**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Use Z = x-u /std dev = 50-45 /8 = 0.625 so probability from chart is 0.7357

As the total estimated time is 1 hr

X = 60-10 = 50

P( can meet demand ) 0.7357

P ( can not meet demand ) : 1- 0.7357 = 0.2676

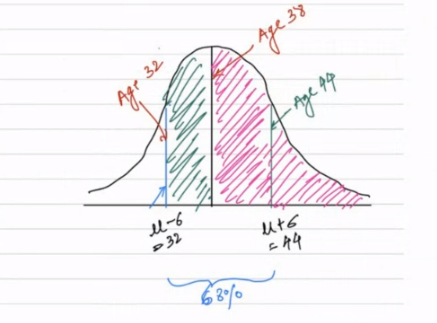
1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 and between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

A is false

As we find z score at age 44 it is 1 and for 38 is 0 which is at the mean

It means 1 std deviation from mean which is 34 % of population between 38 to 44

And more than 44 is 16 %



B is true

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

to analyze the difference between 2 \* X1 and X1 + X2, let's consider their distributions and parameters separately:

For 2 \* X1:

Distribution: Since X1 follows a normal distribution, when multiplied by 2, the resulting variable 2 \* X1 will also follow a normal distribution.

Parameters: The mean of 2 \* X1 would be 2 times the mean of X1, which means the mean of 2 \* X1 is 2μ. The variance of 2 \* X1 would be 4 times the variance of X1, resulting in a variance of 4σ².

For X1 + X2:

Distribution: The sum of two independent normal random variables (X1 and X2) also follows a normal distribution.

Parameters: The mean of X1 + X2 is the sum of the means of X1 and X2, which is μ + μ = 2μ. The variance of X1 + X2 is the sum of the variances of X1 and X2, resulting in a variance of σ² + σ² = 2σ².

In summary:

The distribution of 2 \* X1 is a normal distribution with a mean of 2μ and a variance of 4σ².

The distribution of X1 + X2 is a normal distribution with a mean of 2μ and a variance of 2σ².

It's important to note that these properties hold when X1 and X2 are independent normal random variables. If they are not independent, the resulting distributions and parameters may differ.

In practice, understanding the distributions and parameters of these operations allows us to analyze and make predictions based on the properties of the variables involved.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Sol: Given,

μ = 100

σ = 20

The probability of getting the value b/w a & b is 0.99

So, the value outside the a & b would be 1 – 0.99 = 0.01

And the value outside the a would be 0.01/2 = 0.005

Same the value outside the b would be 0.01/2 = 0.005

Now we are having the probability of a & b and we need to calculate the probability of X – the random variable at a & b having these probabilities

z = x-µ/σ says that x=σ[z] + µ

or Z(-0.005)\*20+100

Z(0.005)\*20+100

z value for probability 0.005 is 2.57

x=σ[z] + µ

= 20 \* 2.57 + 100

= 151.4

x=σ[z] + µ

= 20\* - 2.57 + 100

= 48.6

So, D is the correct option

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Sol: A. Given,

For company 1

μ = 5

σ = 3

For company 2

μ = 7

σ = 4

Total μ = 12 million $

1$ = Rs 45

So, the total would be 12\*45 = Rs 540 million

Total σ = 7

Same 1$ = Rs 45

= 45\* Sqrt of (Variance of profit 1 + Variance of profit 2)

= 45\* Sqrt of (3\*3 + 4\*4)

= 45\*Sqrt of (9+16)

= 45\* sqrt of (25)

= 45\* 5

= Rs 225 million

Probability for annual profit is 95% = 0.95

So, alpha would be 1- 0.95 = 0.05

The z value for 0.05 is 1.65

Then x=σ[z] + µ

= 225\*1.65 + 540

= 911.25

x=σ[-z] + µ

= 225\*-1.65 + 540

= 168.75

So, the range would 911.25 to 168.75

**B.** Specify the 5th percentile of profit (in Rupees) for the company

Sol: Z score for 5th percentile is -1.645

x=σ[z] + µ

= 225\*-1.645 + 540

= 169.88

C. Which of the two divisions has a larger probability of making a loss in a given year?

Sol: Probability for making a loss p(x>0)

For company 1

z = x-µ/σ

= 0- 5/3

= -1.67

= 0.047

For company 2

z = x-µ/σ

= 0- 7/4

= -1.75

= 0.040